R1775

Sub. Code
511101

M.Sc. DEGREE EXAMINATION, NOVEMBER – 2024

First Semester

Mathematics

GROUPS AND RINGS

(CBCS – 2022 onwards)

Time : 3 Hours   Maximum : 75 Marks

Part A $(10 \times 1 = 10)$

Answer **all** the following objective type questions by choosing the correct option.

1. Which of the following is an example of a group under addition? (CO1, K2)

   (a) The set of natural numbers

   (b) The set of integers

   (c) The set of positive integers

   (d) The set of rational numbers excluding zero

2. Which of the following is a necessary property of a subgroup? (CO1, K2)

   (a) It contains the identity element of the group

   (b) It contains at least two elements

   (c) It is closed under multiplication

   (d) It is a finite set

3. Which of the following is a characteristic of normal subgroups? (CO2, K2)

(a) Every subgroup of an abelian group is normal

(b) Normal subgroups must be cyclic

(c) Normal subgroups must be infinite

(d) Normal subgroups must be finite

4. Which of the following is an example of a normal subgroup? (CO2, K2)

(a) $\mathbb{Z}$ in $\mathbb{Q}$

(b) $\mathbb{Z}$ in $\mathbb{R}$

(c) $n\mathbb{Z}$ in $\mathbb{Z}$

(d) $\mathbb{R}$ in $\mathbb{C}$

5. Which property is preserved under direct products of groups? (CO3, K1)

(a) Commutativity

(b) Associativity

(c) Identity elements

(d) All of the above

6. What is a Sylow p-subgroup? (CO3, K2)

(a) A subgroup of prime order

(b) A subgroup whose order is a power of a prime p

(c) A normal subgroup

(d) A subgroup of order p

7. What is a division ring? (CO4, K2)

(a) A ring in which every nonzero element has a multiplicative inverse, but multiplication is not necessarily commutative

(b) A ring where every element has an additive inverse

(c) A ring with no identity element

(d) A commutative ring with zero divisors

8. In the quotient ring R\I what is the additive identity? (CO4, K3)

(a) The element I (b) The element $1 + I$

(c) The coset $0 + I$ (d) The element 0

9. Which of the following is a unit in $\mathbb{Z}$ $[x]$? (CO5, K2)

(a) x (b) $2x + 1$

(c) 1 (d) $x - 1$

10. What is the degree of the polynomial 0 in $\mathbb{Z}$ $[x]$? (CO5, K1)

(a) 0 (b) 1

(c) 2 (d) Undefined

Part B $(5 \times 5 = 25)$

Answer **all** the questions not more than 500 words each.

11. (a) If H is a nonempty finite subset of a group G and H is closed under multiplication then prove that H is a subgroup of G. (CO1, K3)

Or

(b) If H is a subgroup of G, then by the centalizer $C(H)$ of H , $\{x \in G \mid x\, h = h\, x \;\; all \;\; h \in H\}$. Then prove that $C(H)$ is a subgroup of G. (CO1, K3)

12. (a) State and Prove Cauchy's theorem. (CO2, K4)

Or

(b) Prove that the subgroup N of G is a normal subgroup of G then if and only if every left coset of N in G is a right coset of N in G. (CO2, K4)

13. (a) Prove that Conjugacy is an equivalence relation on G. (CO3, K3)

Or

(b) If $o(G) = p^2$ where p is a prime number, then prove that G is abelian. (CO3, K4)

14. (a) If R is a ring with unit element 1 and ϕ is a homomorphism of R onto R' then prove that $\phi(1)$ is the unit element of R'. (CO4, K3)

Or

(b) If ϕ is a homomorphism of R into R' then the kernel $I(\phi)$, then prove that

(i) $I(\phi)$ is a subgroup of R under addition.

(ii) If $a \in I(\phi)$ and $r \in R$ then both ar and ra are in $I(\phi)$. (CO4, K4)

15. (a) Let R be a Euclidean, suppose that for $a, b, c \in R, a \mid bc$ but $(a, b) = 1$, then prove that $a \mid c$. (CO5, K3)

Or

(b) State and prove Division algorithm. (CO5, K4)

Part C $(5 \times 8 = 40)$

Answer **all** the questions not more than 1000 words each.

16. (a) Prove that the Relation $a \equiv b \mod H$ is an equivalence relation. (CO1, K4)

Or

(b) Let G be a finite abelian group in which the number of solution in G of the equation $x^n = e$ is atmost n for every positive integer n. then prove that G must be cyclic group. (CO1, K5)

17. (a) If ϕ is a homomorphism of G into $\overline{G}$ with kernel K, then prove that K is a normal Subgroup of G. (CO2, K4)

Or

(b) State and Prove Sylow's Theorem. (CO2, K5)

18. (a) If p is a prime number and $p^\alpha \mid o(G)$, then prove that G has a subgroup of order P^α. (CO2, K4)

Or

(b) Let G be a group and suppose that G is the internal direct product of $N_1,....N_n$. Let $T = N_1 \times N_2 \times .... \times N_n$, then prove that G and T are isomorphic. (CO2, K4)

19. (a) Prove that if R be a commutative ring with unit element whose ideals are (0) and R itself then R is a field. (CO4, K4)

Or

(b) Prove that if ϕ is a homomorphism of R into R', then

(i) $\phi(0) = 0$

(ii) $\phi(-a) = -\phi(a)$ for every $a \in R$. (CO4, K3)

20. (a) If $f(x)$ and $g(x)$ are primitive polynomials, then prove that $f(x)\,g(x)$ is a primitive polynomials. (CO5, K4)

Or

(b) If R be a Euclidean ring then prove that every element in R is either a unit in R or can be written as the product of a number of prime elements of R. (CO5, K5)

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R1776

Sub. Code
511102

M.Sc. DEGREE EXAMINATION, NOVEMBER – 2024

First Semester

Mathematics

REAL ANALYSIS – I

(CBCS – 2022 onwards)

Time : 3 Hours Maximum : 75 Marks

Part A $(10 \times 1 = 10)$

Answer **all** the following objective type questions by choosing the correct option.

1. Which of the following statements best justifies the claim that if F is closed and K is compact, then $F \cap B$ is compact? (CO1, K1)

   (a) The intersection of a closed set and a compact set is always compact

   (b) The intersection of a closed set and a compact set is closed and bounded

   (c) Closed subsets of compact sets are always compact

   (d) The finite intersection of closed sets and compact sets is always compact

2. Which of the following best defines a compact set? (CO1, K1)

   (a) A set that contains all its limit points

   (b) A set that is closed and bounded

   (c) A set for which every open cover has a finite subcover

   (d) A set that is finite in size

3. Which of the following describes the upper limit of a sequence $\{a_n\}$? (CO2, K1)

(a) The limit of the smallest subsequence

(b) The limit of the largest subsequence

(c) The limit superior, or $\limsup_{n\to\infty} a_n$.

(d) The maximum value of the sequence

4. Which of the following sequences is Cauchy? (CO2, K1)

(a) $a_n = (-1)^n$ (b) $a_n = \frac{1}{n}$

(c) $a_n = n$ (d) $a_n = \sin(n)$

5. Which of the following series does NOT converge absolutely? (CO3, K1)

(a) $\sum_{n=1}^{\infty} \frac{(-1)^n}{n}$ (b) $\sum_{n=1}^{\infty} \frac{1}{n^2}$

(c) $\sum_{n=1}^{\infty} \frac{1}{2^n}$ (d) $\sum_{n=1}^{\infty} \frac{(-1)^n}{n^2}$

6. Which of the following is the value of e correct to three decimal places? (CO3, K1)

(a) 2.718 (b) 3.142

(c) 1.618 (d) 1.414

7. Which of the following statements about monotone functions is true? (CO4, K1)

(a) A function is monotone if and only if it is continuous

(b) A function is monotone if and only if its derivative is nonnegative or nonpositive everywhere

(c) A function is monotone if and only if it is differentiable

(d) A function is monotone if and only if it is bounded

8. Which of the following is true for a connected set in $\mathbb{R}$? (CO4, K1)

(a) Every connected set is an interval

(b) Every interval is connected

(c) Every Connected set is bounded

(d) Every Connected set is Closed

9. Let $f(x)=\frac{1}{x^2}$. What is the nth derivative of $f(x)$ with respect to x? (CO5, K1)

(a) $\frac{n!}{x^{n+1}}$ (b) $\frac{(-1)^n.n!}{x^{n+1}}$

(c) $\frac{(-1)^{n-1}.n!}{x^{n+1}}$ (d) $\frac{(-1)^{n+1}.n!}{x^{n+1}}$

10. Evaluate $\lim_{x\to 0}\frac{\sin(x)}{x}$ using L'Hôpital's Rule. (CO5, K1)

(a) $\lim_{x\to 0}\frac{\sin(x)}{x}=0$ (b) $\lim_{x\to 0}\frac{\sin(x)}{x}=1$

(c) $\lim_{x\to 0}\frac{\sin(x)}{x}=\infty$ (d) $\lim_{x\to 0}\frac{\sin(x)}{x}=\text{undefined}$

Part B $(5 \times 5 = 25)$

Answer **all** the following questions not more than 500 words each.

11\. (a) Let $\{E_n\}, n = 1, 2, 3, \cdots$ be a sequence of countable sets and put $S = \cup_{i=1}^{\infty} E_n$. Then prove that S is countable. (CO1, K2)

Or

(b) Let P be a nonempty perfect set in R^k. Then prove that P is uncountable. (CO1, K2)

12\. (a) Prove that if $P > 1$, $\sum_{n=2}^{\infty} \frac{1}{n(\log n)^p}$ converges, if $p \leq 1$ the series diverages. (CO2, K2)

Or

(b) Suppose $\{s_n\}$ is monotonic. Then prove that $\{s_n\}$ converges iff it is bounded. (CO2, K2)

13\. (a) Prove that e is irrational. (CO3, K1)

Or

(b) For any sequence $\{c_n\}$ of positive numbers, then prove that (CO3, K1)

(i) $\liminf_{n \to \infty} \frac{c_{n+1}}{c_n} \leq \liminf_{n \to \infty} \sqrt[n]{c_n}$

(ii) $\limsup_{n \to \infty} \sqrt[n]{c_n} \leq \limsup_{n \to \infty} \frac{c_{n+1}}{c_n}$

14. (a) Suppose f is a continuous mapping of a compact metric space X into a metric space Y. Then prove that $f(X)$ is compact. (CO4, K2)

Or

(b) If f is a continuous mapping of a metric space X into a metric space Y and if E is a connected subset of X then prove that $f(E)$ is connected. (CO4, K2)

15. (a) Suppose f is continuous on $[a,b]$, $f'(x)$ exists at some point $x \in [a,b], g$ is defined on an interval I which contains the range of f and g is differentiable at the point $f(x)$. If $h(t) = g(f(t))$ $(a \leq t \leq b)$, then prove that h is differentiable at x and $h'(x) = g'(f(x))f'(x)$. (CO5, K2)

Or

(b) Suppose f is a real differentiable function on $[a,b]$ and suppose $f'(a) < \lambda < f'(b)$. Then prove that there is a point $x \in (a,b)$ such that $f'(x) = \lambda$. (CO5, K2)

Part C (5 × 8 = 40)

Answer **all** the following questions not more than 1000 words each.

16. (a) If a set E in R^k has one of the following three properties then prove that it has the other two: (CO1, K3)

(i) E is closed and bounded.

(ii) E is compact.

(iii) Every infinite subset of E has a limit point in E

Or

(b) Prove that suppose $K \subset Y \subset X$. Then K is compact relative to X iff K is compact relative to Y. (CO1, K2)

17. (a) Prove the followings: (CO2, K3)

(i) In any metric space X, every convergent sequence is a Cauchy sequence.

(ii) If X is a compact metric space and if $\{p_n\}$ is a Cauchy sequence in X, then $\{p_n\}$ converges to some point of X.

(iii) In R^k, every Cauchy sequence converges.

Or

(b) (i) Suppose $a_1 \geq a_2 \geq a_3 \geq \ldots \geq 0$. Then prove that the series converges if and only if the series $\sum_{k=0}^{\infty} 2^k a_{2^k} = a_1 + 2a_2 + 4a_4 + 8a_8 + \ldots$ converges.

(ii) The sub sequential limits of a sequence $\{p_n\}$ in a metric space X from a closed subset of X. (CO2, K2)

18. (a) State and prove root test and ratio test. (CO3, K3)

Or

(b) Let $\sum a_n$ be a series of real numbers which converges but not absolutely. Suppose $-\infty \leq \alpha \leq \beta \leq \infty$. Then prove that there exists a rearrangement $\sum a'_n$ with partial sums s'_n such that $\liminf_{n\to\infty} s'_n = \alpha$, $\limsup_{n\to\infty} s'_n = \beta$. (CO3, K4)

19. (a) (i) Let E be a noncompact set in R^1. Then prove that

(1) there exist a continuous function on E which is not bounded.

(2) there exist a continuous and bounded function on E which has no maximum.

(ii) Let f be monotonic on (a,b). Then prove that the set of points of (a,b) at which f is discontinuous is atmost countable. (CO4, K3)

Or

(b) Let f be a continuous mapping of a compact metric space X into a metric space Y. Then prove that f is uniformly continuous on X. (CO4, K3)

20. (a) State and prove any two mean value theorems. (CO5, K4)

Or

(b) Suppose f and g are real and differentiable in (a,b) and $g'(x) \neq 0$ for all $x \in (a,b)$, where $-\infty \leq a < b \leq +\infty$. Suppose $\frac{f'(x)}{g'(x)} \to A$ as $x \to a$. If $f(x) \to 0$ and $g(x) \to 0$ as $x \to a$, or if $g(x) \to \infty$ as $x \to a$ then prove that $\frac{f(x)}{g(x)} \to A$ as $x \to a$. (CO5, K3)

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R1777

Sub. Code
511103

M.Sc. DEGREE EXAMINATION, NOVEMBER – 2024

First Semester

Mathematics

ORDINARY DIFFERENTIAL EQUATIONS

(CBCS – 2022 onwards)

Time : 3 Hours Maximum : 75 Marks

Section A $(10 \times 1 = 10)$

Answer **all** the following objective type questions by choosing the correct option.

1. Solution of the Ordinary Differential Equation $y'' + 2y' + 5y = e\sin(t)$ when $y(0) = 0$ and $y'(0) = 1$. (Without solving for the constants we get in the partial fractions) (CO1, K2)

   (a) $e^{t}\left[A\cos t + A1\sin t + B\cos(2t) + \frac{(B1)}{2}\sin(2t)\right]$

   (b) $e^{-t}[A\cos t + A1\sin t + B\cos(2t) + B1\sin(2t)]$

   (c) $e^{-t}\left[A\cos t + A1\sin t + B\cos(2t) + \frac{(B1)}{2}\sin(2t)\right]$

   (d) $e^{t}[A\cos t + A1\sin t + B\cos(2t) + (B1)\sin(2t)]$

2. The solution of the differential equation $2x\frac{dy}{dx}-y=0$; $y(1)=2$ represents (CO1, K2)

(a) straight line (b) parabola

(c) circle (d) ellipse

3. The differential equation $i\frac{dv}{dx}=-\frac{x+y}{1+x^2}$ is (CO2, K2)

(a) of variable separable form

(b) homogeneous

(c) linear

(d) of second order

4. The order and degree of differential equation $\sqrt{\sin x}(dx+dy)=\sqrt{\cos x}(dx-dy)$ are ―――――― and ―――――― respectively. (CO2, K3)

(a) 1, 2 (b) 2, 2

(c) 1, 1 (d) 2, not obtained

5. For the differential equation $x^2(1-x)\frac{d^2y}{dx^2}+x\frac{dy}{dx}+y=0$ ―――――― (CO3, K3)

(a) $x=1$ is an ordinary point

(b) $x=1$ is a regular singular point

(c) $x=0$ is an irregular singular point

(d) $x=0$ is an ordinary point

6. The number of arbitrary constants in the particular solution of a differential equation of fourth order is ―――――― (CO3, K2)

(a) 1 (b) 0

(c) 2 (d) 4

7. Consider the initial vaule problem $Y'(t) = f(t)y(t)$ with $y(0) = 1$ where $f : R \to R$ is a continuos function. Then this initial value problem has ———— (CO4, K2)

(a) infinitely many solutions for some f

(b) a unique solution in R

(c) no solution in R for some f

(d) a solution in an interval containing 0, but not on R for some f

8. If a second order differential equation has the form $y'' = f(t, y')$, then $v = y'$ satisfies the first order equation ———— (CO4, K3)

(a) $v' = f(t, v)$ (b) $v = f(t, v)$

(c) $v'' = f(t, v)$ (d) $v'' = 0$

9. Exact differential equations can be rewritten as a total derivative of a function, called a ———— (CO5, K2)

(a) exact (b) first order equation

(c) potential function (d) none

10. The order of the differential equation of family of circles which passes through origin and whose centre is on y - axis is ———— (CO5, K2)

(a) 1 (b) 2

(c) 3 (d) 4

Section B (5 × 5 = 25)

Answer **all** the following questions not more than 500 words each.

11. (a) If ϕ_1, ϕ_2 are two solutions of $L(y) = 0$ on an interval I containing a point x_0 then show that $W(\phi_1, \phi_2)(x_0) = e^{-\alpha_1(x - x_0)} W(\phi_1, \phi_2)(x_0)$. (CO1, K5)

Or

(b) Let ϕ_1, ϕ_2 be two linearly independent solutions of $L(y) = 0$ on an interval I. Prove that every solution ϕ of $L(Y) = 0$ can be written uniquely as $\phi = c_1 \phi_1 + c_2 \phi_2$ where $c_1 c_2$ are constants. (CO1, K4)

12. (a) If ϕ_1 is a solution of $L(y) = y'' + a_1(x)y' + a_2(x)y = 0$ on an interval I, and $\phi_1(x) \neq 0$ on I, a second solution ϕ_2 of $L(y) = y'' + a_1(x)y' + a_2(x)y = 0$ on I is given by

$$\phi_2(x) = \phi_1(x) \int_{x_0}^{x} \left[\frac{1}{\phi_1(s)}\right]^2 \exp\left[-\int_{x_0}^{s} a_1(t)\,dt\right] ds.$$

Show that the functions ϕ_1, ϕ_2 form a basis for the solution on I.

(CO2, K3)

Or

(b) Find two linearly independent solution of the equation $(3x-1)^2 y'' + (9x-3)y' - 9y = 0$ for $x > \frac{1}{3}$.

(CO2, K5)

13. (a) Find all solution of the following equations for $|x| > 0$: (CO3, K4)

(i) $x^2 y'' + xy' + 4y = 1$

(ii) $x^2 y'' - 3xy' + 5y = 0$.

Or

(b) Compute the indicial polynomials, and their roots, for the equation $x^2 y'' + xy' + \left(x^2 - \frac{1}{4}\right) y = 0$.

(CO3, K3)

14. (a) Suppose S is either a rectange $|x - x_0| \leqq a$, $|y - y_0| \leqq b$ $(a, b > 0)$ or a strip $|x - x_0| \leqq a$, $|y - y_0| < \infty$, $(a > 0)$ and that f is a real-valued function defined on S such that $\partial f / \partial y$ exists, is continuous on S, and $\left|\frac{\partial f}{\partial y}(x, y)\right| \leq K, ((x, y) in S)$ for some $K > 0$. Then prove that f satisfies Lipschitz condition on S with Lipschitz constant K. (CO4, K3)

Or

(b) Find the real valued solution of $y' = \frac{x + x^2}{y - y^2}$.

(CO4, K5)

15. (a) Let f be a continuous vector valued function defined on $R: |x - x_0| \leqq a$, $|y - y_0| \leqq b$ $(a, b > 0)$ and suppose f satisfies a Lipschitz condition on R. Prove that if M is a constant such that $f|(x, y)| \leqq M$ for all (x, y) in R, the successive approximations $\{\phi_k\}$, $k = 0, 1, 2$ given by $\phi_0(x) = y_0$ converge on the internal $I: |x - x_0| \leqq \alpha$ = minimum $\{a, b / M\}$, to a solution ϕ of the initial value problem $y' = f(x, y)$, $y(x_0) = y_0$ on I. (CO5, K4)

Or

(b) Let $f(x, y) = \frac{\cos y}{1 - x^2}$, $|x| < 1$. Show that f satisfies a Lipschitz condition on every strip $S_n : |x| \leqq a$, where $0 < a < 1$. (CO5, K6)

Section C (5 × 8 = 40)

Answer **all** the following questions not more than 1,000 words each.

16. (a) Consider the equation with constant coefficients $L(y) = P(x)e^{ax}$, where P is the polynomial given by $P(x) = b_0x^m + b_1x^{m-1} + ... + b_m$, $(b_0) \neq 0$. Suppose a is a root of the characteristic polynomial p of L of multiplicity j. Then prove that there is a unique solution of ψ of $L(y) = P(x)e^{ax}$ of the form $\psi(x) = x^j(c_0x^m + c_1x^{m-1} + ... + c_m)e^{ax}$, where $c_0, c_1, ..., c_m$ are constants determined by the annihilator method. (CO1, K2)

Or

(b) Show that every solution of the constant coefficient equation $y'' + a_1y' + a_2y = 0$ is bounded on $0 \leq x > \infty$ if and only if the real parts of the roots of the characteristic polynomial are non positive and roots with zero real part have multiplicity one. (CO1, K2)

17. (a) State and prove Existence Theorem for Analytic Coefficients. (CO2, K4)

Or

(b) Computer the solution ϕ of $y''' - xy = 0$ which satisfies $\phi(0) = 1$, $\phi'(0) = 0$, $\phi''(0) = 0$. (CO2, K4)

18. (a) (i) Show that –1 and 1 are regular singular points for the Legendre equation $(1 - x^2)y'' - 2xy' + \alpha(\alpha + 1)y = 0$.

(ii) Find the indicial polynomial and its roots corresponding to the point $x = 1$. (CO3, K6)

Or

(b) Derive the Bessel's function of zero order of the second kind K_0. (CO3, K6)

19. (a) Show that the successive approximations ϕ_k, defined by $\phi_0(x) = y_0$ exists as continuous functions on $I : |x - x_0| \leqq \alpha$ = minimum $\{a, b/M\}$, and $(x, \phi_k(x))$ is in R for x in 1. Indeed, the ϕ_k satisfy $|\phi_k(x) - y_0| \leqq M|x - x_0|$ for all x in I. (CO4, K3)

Or

(b) (i) Find the solution of $y' = 2y^{1/2}$ passing through the point (x_0, y_0) where $y_0 > 0$.

(ii) Find all solutions of this equation passing through $(x_0, 0)$. (CO4, K3)

20. (a) Consider the equation $y' = f(x)p(\cos y) + g(x)q(\sin y)$ where *f*, *g* are continuous for all real *x*, and *p, q* are polynomials. Show that every initial value problem for this equation has a solution which exists for all real *x*. (CO5, K5)

Or

(b) Let *f* be a real-value continous function of the strip $S : |x - x_0| \leq a$, $|y| < \infty$, $(a > 0)$ and suppose that f' satisfies a Lipschitz condition on S with Lipschitz constant $K > 0$. Then show that the successive approximations $\{\phi_k\}$ for the problem $y' = f(x, y)$, $y(x_0) = y_0$ exist on the entire interval $|x - x_0| \leq a$, and converges there to a solution ϕ. (CO5, K5)

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R1778

Sub. Code
511104

M.Sc. DEGREE EXAMINATION, NOVEMBER – 2024

First Semester

Mathematics

ANALYTIC NUMBER THEORY

(CBCS – 2022 onwards)

Time : 3 Hours Maximum : 75 Marks

Section A $(10 \times 1 = 10)$

Answer **all** the following objective type questions by choosing the correct option.

1. If $n = 5$ then the value of $\phi(n)$ is ——————— (CO1, K1)

   (a) 1 (b) 3

   (c) 4 (d) 2

2. If $n = 10$ then the value of $\mu(n)$ is ——————— (CO1, K1)

   (a) 1 (b) 0

   (c) –1 (d) 2

3. Choose Riemann zeta function ——————— (CO2, K2)

   (a) $\xi(s) = \sum_{n=0}^{\infty} \frac{1}{n^s}$ if $s > 1$

   (b) $\xi(s) = \sum_{n=1}^{\infty} \frac{1}{n^s}$ if $s > 1$

   (c) $\xi(s) = \sum_{n=1}^{\infty} \frac{1}{n}$ if $s > 1$

   (d) $\xi(s) = \sum_{n=1}^{\infty} \frac{1}{n^s}$ if $s \leq 1$

4. The series $\sum_{n=1}^{\infty} \frac{\mu(n)}{n^2}$ is ———— (CO2, K1)

(a) Convergent (b) Absolutely Convergent

(c) Divergent (d) Oscillates

5. Theorems relating different weighted ———— of the same function are called Tauberian theorems. (CO3, K1)

(a) averages (b) sum

(c) difference (d) composite

6. Choose Chebyshev's ϑ–function equation ———— (CO3, K1)

(a) $\vartheta(x) = \sum_{p \le x} \log x$ (b) $\vartheta(x) = \sum_{p < x} \log p$

(c) $\vartheta(x) = \sum_{p \le x} \log p$ (d) $\vartheta(p) = \sum_{p \le x} \log p$

7. The linear congruence $2x \equiv 3 \pmod 4$ has ———— (CO4, K1)

(a) unique solution (b) no solutions

(c) many solution (d) zero

8. No prime $p \equiv 3 \pmod 4$ is the ———— (CO4, K1)

(a) divisibility of two squares

(b) difference of two squares

(c) product of two squares

(d) sum of two squares

9. Legendre's symbol $(n \mid p)$ is a completely ———— function of n. (CO5, K1)

(a) additive (b) commutative

(c) multiplicative (d) modulo

10. 219 is a quadratic residue of ———— (CO5, K1)

(a) mod 384 (b) mod 381

(c) mod 385 (d) mod 383

Section B $(5 \times 5 = 25)$

Answer **all** the following questions not more than 500 words each.

11. (a) Show that the given integers a and b with $b > 0$, there exists a unique pair of integers q and r such that $a = bq + r$. with $0 \le r \le b$. Moreover $r = 0$ if $b \mid a$ (CO1, K5)

Or

(b) Prove that if $n \ge 1$ we have $\sum_{d \mid n} \phi(d) = n$ (CO1, K4)

12. (a) State and prove Euler's summation formula. (CO2, K3)

Or

(b) Show that if $x \ge 1$ and $\alpha > 0, \alpha \ne 1$, we have

$$\sum_{n \le x} \sigma_\alpha(n) = \frac{\zeta(\alpha+1)}{\alpha+1} x^{\alpha+1} + O\left(x^\beta\right) \text{ where } \beta = \max\{1, \alpha\}$$

(CO2, K4)

13. (a) Show that $\lim_{x\to\infty}\left(\frac{M(x)}{x}-\frac{H(x)}{x\log x}\right)=0$ (CO3, K3)

Or

(b) State and prove Abel's identity. (CO3, K4)

14. (a) State and prove Euler-Fermat theorem. (CO4, K3)

Or

(b) Solve the following congruences: (CO4, K4)

(i) $5x \equiv 3 \pmod{24}$

(ii) $25x \equiv 15 \pmod{120}$

15. (a) Determine those odd primes p for which 3 is a quadratic residue and those for which it is a nonresidue. (CO5, K3)

Or

(b) If P and Q are positive odd integers with $(P, Q)=1$, then prove that $(P \mid Q)\,(Q \mid P)=(-1)^{\frac{(P-1)(Q-1)}{4}}$ (CO5, K4)

Section C (5 × 8 = 40)

Answer **all** the following questions not more than 1000 words each.

16. (a) State and prove Fundamental theorem of arithmetic. (CO1, K6)

Or

(b) Prove that for $n \geq 1$ we have $\phi(n)=n\prod_{p\mid n}\left(1-\frac{1}{p}\right)$ (CO1, K5)

17. (a) Prove that if $x \geq 1$ we have (CO2, K6)

(i) $\sum_{n \leq x} \frac{1}{n} = \log x + C + O\left(\frac{1}{x}\right)$

(ii) $\sum_{n \leq x} \frac{1}{n^s} = \frac{x^{1-s}}{1-s} + \zeta(s) + O(x^{-s})$ if $s > 0, s \neq 1$

(iii) $\sum_{n > x} = O(x^{1-s})$ if s > 1

(iv) $\sum_{n \leq x} n^{\alpha} \frac{x^{\alpha+1}}{\alpha+1} + O(x^{\alpha})$ if $\alpha \geq 0$

Or

(b) Prove that for all $x \geq 1$ we have $\sum_{n \leq x} d(n) = x \log x + (2C-1)x + O(\sqrt{x})$, where C is Euler's Constant (CO2, K5)

18. (a) State and prove Shapiro's Tauberian theorem. (CO3, K6)

Or

(b) Prove that for every integer $n \geq 2$ we have $\frac{1}{6}\frac{n}{\log n} < \pi(n) < 6\frac{n}{\log n}$ (CO3, K6)

19. (a) State and prove Wolstenholnie's theorem. (CO4, K5)

Or

(b) Given integers r, d and k such that $d \mid k,\ d > 0,\ k \geq 1$ and $(r, d) = 1$. Then show that the number of elements in the set $S = \{r + td : t = 1, 2, \ldots, k/d\}$ which are relatively prime to k is $\varphi(k)/\varphi(d)$. (CO4, K6)

20. (a) (i) State and prove Reciprocity jaw for Jacobi symbols. (CO5, K5)

(ii) Determine whether 888 is a quadratic residue or nonresidue of the prime 1999.

Or

(b) State and prove Gauss' lemma. (CO5, K6)

———————

R1779

Sub. Code
511505

M.Sc. DEGREE EXAMINATION, NOVEMBER – 2024

First Semester

Mathematics

Elective : OBJECT ORIENTED PROGRAMMING AND C++

(CBCS – 2022 onwards)

Time : 3 Hours Maximum : 75 Marks

Part A $(10 \times 1 = 10)$

Answer **all** the following objective questions by choosing the correct option.

1. Memory release operator is known as ————. (CO1, K1)

   (a) Delete (b) New

   (c) Setw (d) Endl

2. A typical C++ program would contain ———— sections (CO1, K1)

   (a) 3 (b) 4

   (c) 2 (d) 5

3. Which keyword is used to make a member function accessible outside (CO2, K2)

   (a) Public (b) Private

   (c) Protected (d) Friend

4. What happens if you try to access a non-static data member from inside a static member function in C++? (CO2, K2)

   (a) It generates a compile-time error

   (b) It generates a runtime error

   (c) It accesses the member normally

   (d) It accesses the member using the 'this' pointer

5. How do you declare a pointer to an integer variable ptr in C++? (CO3, K3)

   (a) int *ptr; (b) int ptr;

   (c) ptr int *; (d) pointer ptr;

6. How do you define a copy constructor? (CO3, K3)

   (a) *ClassName()* {...}

   (b) *Class Name(const ClassName and obj)* {...}

   (c) *Copy (ClassName obj)* {...}

   (d) *ClassName (const ClassName obj)* {...}

7. How does function overloading contribute to compile time polymorphism? (CO4, K4)

   (a) It allows a function to be called with different types of arguments

   (b) It allows a derived class to provide a specific implementation of a base class function

   (c) It allows the compiler to select the appropriate function based on the number or types of arguments

   (d) It allows a function to be defined in different source files

8. Which of the following unary operators cannot be overloaded? (CO4, K4)

   (a) + (b) -

   (c) * (d) .

9. What is inheritance in C++? (CO5, K5)

   (a) The ability to derive a new class from an existing class

   (b) The ability to create objects from classes

   (c) The ability to define private members in a class

   (d) The ability to override member functions

10. How does a pure virtual function differ from a regular virtual functions? (CO5, K5)

(a) A pure virtual function has no implementation

(b) A pure virtual function cannot be overridden

(c) A pure virtual function must be defined in every derived class

(d) A pure virtual function is static

Part B (5 × 5 = 25)

Answer **all** the questions not more than 500 words each.

11. (a) State the declaration of variables. (CO1, K1)

Or

(b) List all the derived data types. (CO1, K1)

12. (a) How do the Static member function work? (CO2, K2)

Or

(b) Discuss a simple class with an example. (CO2, K2)

13. (a) Define String with an example. (CO3, K3)

Or

(b) Discuss the copy constructor with an example. (CO3, K3)

14. (a) State the compile time polymorphism. (CO4, K4)

Or

(b) Discuss the function overloading. (CO4, K4)

15. (a) Describe the derived class in C++. (CO5, K5)

Or

(b) Explain the pure virtual inheritance. (CO5, K5)

Part C (5 × 8 = 40)

Answer **all** the questions not more than 1,000 words each.

16. (a) Discuss the most commonly used Manipulators. (CO1, K1)

Or

(b) List all the operators in C++. (CO1, K1)

17. (a) Elaborate how Friend and virtual functions work? (CO2, K2)

Or

(b) Explain Constructor and destructor. (CO2, K2)

18. (a) Define Strings and explain the Dynamic constructors. (CO3, K3)

Or

(b) Explain *this* pointer. (CO3, K3)

19. (a) Explain the Overloading in unary and binary operators. (CO4, K4)

Or

(b) Discuss operator overloading. (CO4, K4)

20. (a) Describe the types of Inheritance. (CO5, K5)

Or

(b) Explain how the virtual function work in C++. (CO5, K5)

———————

R1780

Sub. Code
511301

M.Sc. DEGREE EXAMINATION, NOVEMBER – 2024

Third Semester

Mathematics

CLASSICAL DYNAMICS

(CBCS – 2022 onwards)

Time : 3 Hours Maximum : 75 Marks

Part A $(10 \times 1 = 10)$

Answer **all** the following objective type questions by choosing the correct option.

1. The sum of all the forces, real and inertial, acting on each particle of the system is equal to (CO1, K2)

   (a) Zero (b) Constant

   (c) Greater than zero (d) Less than zero

2. The system is ———— if any of the constraint equations or the transformation equations contain time explicitly. (CO1, K2)

   (a) Scleronomic (b) Rheonomic

   (c) Nonholonomic (d) Holonomic

3. The derivation of Lagrange's equation for a holonomic system required that the generalized coordinates be ———— (CO2, K3)

   (a) Dependent (b) Constraints

   (c) Ske-symmetric (d) Independent

4. A holonomic conservative system with $T_1 \neq 0$ is (CO2, K3)

(a) Gyroscopic system

(b) Rheonomic system

(c) Natural system

(d) Non holonomic systems

5. The ———————— is one of the classical problem of the calculus of variation. (CO3, K6)

(a) Definite integral problem

(b) Keplar Problem

(c) Brachistochrone problem

(d) None of the above

6. Jacobi's form of the principle of least action is (CO3, K6)

(a) $\delta A = \int \sqrt{2(h+V)}ds = 0$

(b) $\delta A = \int \sqrt{2(h-V)}ds = 0$

(c) $\delta A = \delta \int \sqrt{2(h+V)}ds = 0$

(d) $\delta A = \delta \int \sqrt{2(h-V)}ds = 0$

7. The function $S(q_0, q_1, t_0, t_1)$ is assumed to be twice differentiable in all its arguments and is known as (CO4, K3)

(a) Hamilton's principal function

(b) Pfaffian differential form

(c) Lagrange problem

(d) Jacobi Equation

8. Choose the modified Hamilton-Jacobi equation __________ (CO4, K3)

(a) $H\left(W, \frac{\partial W}{\partial q}\right) = \alpha_n$ (b) $H\left(q, \frac{\partial W}{\partial q}\right) = \alpha_W$

(c) $H\left(q, \frac{\partial W}{\partial q}\right) = \alpha_n$ (d) $H\left(q, \frac{\partial W}{\partial q}\right) \neq \alpha_n$

9. A transformation from (q, p) to (Q, P) which preserves the canonical form of the equation of the motion is known as (CO5, K2)

(a) Functional transformation

(b) Canonical transformation

(c) Non-functional transformation

(d) Contact transformation

10. The generating function ϕ is not an explicit function of __________ (CO5, K2)

(a) ε (b) τ

(c) $\partial\tau$ (d) λ

Part B $(5 \times 5 = 25)$

Answer **all** the questions not more than 500 words each.

11. (a) Explain Configuration space. (CO1, K2)

Or

(b) Explain Nonholonomic Constraints. (CO1, K2)

12. (a) Derive the standard form of Lagrange's equation for a nonholonomic system. (CO2, K3)

Or

(b) Explain the Kepler problem in terms of polar coordinates. (CO2, K3)

13. (a) State the brachistochrone problem and solve it. (CO3, K6)

Or

(b) A particle of mass m is attracted to a fixed point O by an inverse square force, that is $F_r = \frac{\mu m}{r^2}$ where μ is the gravitational coefficient. Using the plane polar coordinates (r, θ) to describe the position of the particle, find the equation of motion. (CO3, K6)

14. (a) Derive Hamilton's canonical equations. (CO4, K3)

Or

(b) Use the Hamilton-Jacobi method to analyse the Keplar Problem. (CO4, K3)

15. (a) Derive the Homogeneous Canonical Transformations. (CO5, K2)

Or

(b) Consider the transformation $Q\sqrt{e^{-2q} - p^2}$, $P = \cos^{-1}(pe^q)$ Use the Poisson bracket to show that it is canonical. (CO5, K2)

Part C (5 × 8 = 40)

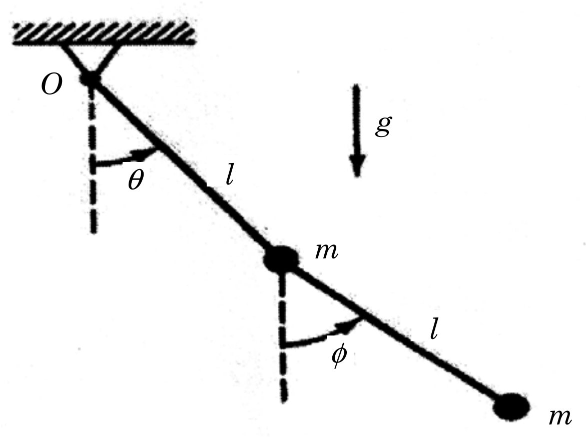
Answer **all** the questions not more than 1000 words each.

16. (a) A particle of mass m is suspended by a massless wire of length $r = a + b\cos\omega t$ $(a > b > 0)$ to form a spherical pendulum. Find the equation of motion. (CO1, K2)

Or

(b) Explain Generalized Momentum and Angular Momentum. (CO1, K2)

17. (a) A double pendulum consists of two particles suspended by massless rods, as shown in below. Assuming that all motions takes place in a vertical plane, find the differential equations of motions. (CO2, K3)



Or

(b) Explain the Routhian function with the illustration. (CO2, K3)

18. (a) Find the equation of motion for a charged particle in an electromagnetic field by using the Hamilton's equation. (CO3, K6)

Or

(b) Derive the Jacobi's form of the principal of least action. (CO3, K6)

19. (a) Derive pfaffian Differential forms. (CO4, K3)

Or

(b) State and prove Stackel' s Theorem. (CO4, K3)

20. (a) Consider a rheonomic transformation, $\begin{matrix} Q = \sqrt{2qe^t}\cos p \\ P = \sqrt{2q^{-1}}\sin p \end{matrix}$ Show that the transformation is canonical. (CO5, K2)

Or

(b) Consider the transformation $\begin{matrix} Q = q - tp + \frac{1}{2}gt^2 \\ P = p - gt \end{matrix}$.

Find $K - H$ and the generating functions. (CO5, K2)

———————

R1781

Sub. Code
511302

M.Sc. DEGREE EXAMINATION, NOVEMBER – 2024

Third Semester

Mathematics

TOPOLOGY

(CBCS – 2022 onwards)

Time : 3 Hours | Maximum : 75 Marks

Part A $(10 \times 1 = 10)$

Answer **all** the following objective questions by choosing the correct option.

1. Let Y be a subspace of X. If U is open in Y and Y is open in X then U is ———— in X. (CO1, K2)

   (a) open

   (b) closed

   (c) upper bound

   (d) convex

2. Let X be a topological space then the ———— of closed sets are closed. (CO1, K1)

   (a) intersection.

   (b) arbitrary intersection

   (c) arbitrary union

   (d) union

3. If each space X_α is a Hausdorff space, then $\prod X_\alpha$ is a Hausdorff space in ———— topologies. (CO2, K2)

(a) box

(b) both box and product

(c) either box and product

(d) product

4. The image of a connected space under a continuous map is ———— (CO2, K2)

(a) connected (b) open

(c) closed (d) continuous

5. Every closed subspace of a compact space is ———— (CO3, K2)

(a) closed (b) compact

(c) open (d) cover

6. A space X is homeomorphic to an open subspace of a compact Hausdorff space if and only if X is ———— (CO3, K1)

(a) compact

(b) connected

(c) locally, compact Hausdorff

(d) hausdorff.

7. If a space X has a countable basis for its topology, then X is said to be ———— (CO4, K1)

(a) first- countable (b) second- countable

(c) dense (d) separable

8. The space R_k is Hausdorff and ———— (CO4, K2)

(a) regular (b) not normal

(c) not regular (d) not dense

9. A 1-manifold is called ———— (CO5, K1)

(a) curve (b) surface

(c) support (d) metrizable

10. Let X be metrizable. Then X is ———— under every metric that gives the topology of X. (CO5, K1)

(a) bounded (b) closed

(c) open (d) regular

Part B $(5 \times 5 = 25)$

Answer **all** the following questions in not more than 500 words each.

11. (a) Show that the topologies of R_l and R_k are strictly finer than the standard topology on R. But are not comparable with one another. (CO1, K2)

Or

(b) Prove that every finite point set in a Hausdorff space X is closed. (CO1, K5)

12. (a) State and prove the sequence lemma. (CO2, K3)

Or

(b) Prove that the union of a collection of connected subspaces of that have a point in common is connected. (CO2, K5)

13. (a) State and prove the extreme value theorem. (CO3, K3)

Or

(b) Prove that let Y be a subspace of X. then Y is compact if and only if every covering of Y by sets open in X contains a finite subcollection covering Y. (CO3, K5)

14. (a) Show that let X be a topological space. Let one-point sets in be closed, then X is regular if and only if given a point x of X and a neighborhood U of x, there is a neighborhood V of x such that $\overline{V} \subset U$. (CO4, K2)

Or

(b) Prove that every metrizable space is normal. (CO4, K5)

15. (a) Show that let X be a set; let $\mathfrak{D}$ be a collection of subsets of X that is maximal with respect to the finite intersection property. Then any finite intersection of elements of $\mathfrak{D}$ is an element of $\mathfrak{D}$. (CO5, K2)

Or

(b) Prove that let $A \subset X$; let $f : A \to Z$ be a continuous map of A into the Hausdorff space Z. There is at most one extension of f to a continuous function $g : \overline{A} \to Z$. (CO5, K5)

Part C $(5 \times 8 = 40)$

Answer **all** the following questions in not more than 1000 words each.

16. (a) Prove that let X be a space satisfying the T_1 axiom; let A be a subset of X. Then the point x is a limit point of A if and only if every neighborhood of x contains infinitely many points. (CO1, K5)

Or

(b) Show that let $f : A \to X \times Y$ be a given by the equation $f(a) = (f_1(a), f_2(a))$. Then f is continuous if and only if the functions $f_1 : A \to X$ and $f_2 : A \to Y$ are continuous. (CO1, K2)

17. (a) Prove that the topologies on R^n induced by the euclidean metric d and the square metric ρ are the same as the product topology on R^n. (CO2, K5)

Or

(b) Show that if L is a linear continuum in the order topology, then L is connected and so are intervals and rays in L. (CO2, K2)

18. (a) Prove that let X be a nonempty compact Hausdorff space. If X has no isolated points, then X is uncountable. (CO3, K5)

Or

(b) Let X be a metrizable space. Then show that the following are equivalent: (CO3, K2)

(i) X is compact.

(ii) X is limit point compact.

(iii) X is sequentially compact.

19. (a) State and prove the Urysohn lemma. (CO4, K5)

Or

(b) Prove that every regular space with a countable basis is normal. (CO4, K5)

20. (a) State and prove the Tychonoff theorm. (CO5, K5)

Or

(b) Show that let X be a completely regular space. There exists a compactification Y of X having the property that every bounded continuous map $f : X \to R$ extends uniquely to a continuous map of Y into R. (CO5, K2)

———————

R1782

Sub. Code
511303

M.Sc. DEGREE EXAMINATION, NOVEMBER – 2024

Third Semester

Mathematics

CALCULUS OF VARIATIONS AND INTEGRAL EQUATIONS

(CBCS – 2022 onwards)

Time : 3 Hours Maximum : 75 Marks

Part A $(10 \times 1 = 10)$

Answer **all** the following objective questions by choosing the correct option.

1. The Characteristic values of λ are ———— the squares of the semiaxes. (CO1, K1)

   (a) directly proportional to

   (b) inversely proportional to

   (c) equal to

   (d) approximately equal to

2. The symbol of Lagrange's multipliers is ————. (CO1, K1)

   (a) μ (b) α

   (c) β (d) λ

3. In one-dimensional calculus, what do we call a maximum and minimum point collectively? (CO2, K2)

   (a) Optimum (b) Turning point

   (c) Extremum (d) Saddle point

4. The Euler-Lagrange equation is used to find extrema of: (CO2, K2)

(a) Functions (b) Functionals

(c) Vectors (d) Constants

5. What mathematical property does the Hankel transform satisfy that is analogous to the Fourier transform? (CO3,K3)

(a) Symmetry (b) Orthogonality

(c) Linearity (d) Commutativity

6. The Hankel transform of $\frac{r^2 d}{dr}$ is ————. (CO3,K3)

(a) $-\frac{2d}{dk}$ (b) $2\frac{d}{dk}$

(c) $-2k$ (d) $2k$

7. The process of reducing a Volterra integral equation to a system of algebraic equations often requires ————. (CO4, K4)

(a) Solving a differential equation

(b) The method of successive approximations

(c) Inverting a matrix

(d) Solving a series expansion

8. The solution to a linear integral equation with a convolution integral often requires ————. (CO4, K4)

(a) Solving a differential equation

(b) Inverting a matrix

(c) Applying an integral transform

(d) Solving a Fredholm integral equation

9. In Volterra integral equations, $K(x, t)$ represents ——————. (CO5, K5)

(a) The solution function

(b) The kernel function

(c) The integral bounds

(d) The differential operator

10. What is the necessary and sufficient condition for the existence of a solution to a linear integral equation according to Fredholm's First theorem? (CO5, K5)

(a) The kernel must be symmetric

(b) The kernel must be continuous

(c) The Fredholm's determinant must be nonzero

(d) The integral equation is zero

Part B $(5 \times 5 = 25)$

Answer **all** the questions not more than 500 words each.

11. (a) Calculate $I(x)$ and $I(\cosh x)$, if $I(y) = \int_0^1 \sqrt{1 + y'^2}\, dx$. (CO1, K1)

Or

(b) Obtain the Euler equation relevant to the determination of extremals of the integral $\int_0^1 F(x, y, y')\, dx$, if $F = xy'^2 - yy' + y$. (CO1, K1)

12. (a) Find the shortest smooth plane curve joining two distinct points (x_1, y_1) and (x_2, y_2). (CO2, K2)

Or

(b) Find the extremals of the functional

$$\int_0^{\frac{\pi}{2}} ((y')^2 - y^2 + x^2 y)\, dx \text{ with } y(0) = -1 \text{ and } y\left(\frac{\pi}{2}\right) = -1.$$

(CO2, K2)

13. (a) Evaluate $\int_0^{\infty} r\left\{\frac{d^2 f}{dr^2} + \frac{1}{r}\frac{df}{dr}\right\} J_0(px)\, dx$ where $f(r) = \frac{e^{-ax}}{r}$. (CO3, K3)

Or

(b) Find the Hankel transform $\frac{\sin ax}{x}$ taking $xJ_0\, P(x)$ as the kernal. (CO3, K3)

14. (a) Invert the integral equation

$$g(s) = f(s) + \lambda \int_0^{2\pi} (\sin s \cos t)\, g(t)\, dt.$$

(CO4, K4)

Or

(b) State and prove Fredholm Alternative Theorem. (CO4, K4)

15. (a) Find the Neumann series for the solution of the integral equation $g(s) = (1+s) + \lambda \int_0^{s} (s-t)\, g(t)\, dt$. (CO5, K5)

Or

(b) Solve the Fredholm integral equation $g(s) = e^s - \frac{1}{2}e + \frac{1}{2} + \frac{1}{2}\int_0^1 g(t)\, dt$ by the method of successive approximations. (CO5, K5)

Part C $(5 \times 8 = 40)$

Answer **all** the questions not more than 1000 words each.

16. (a) Prove that Dirichlet problem is equivalent to variational problem. (CO1, K1)

Or

(b) Prove that the derivative of the variation with respect to an independent variable is the same as the variation of the derivative. (CO1, K1)

17. (a) Explain Brachistochrone problem. (CO2, K2)

Or

(b) A geodesic on a given surface is a curve, lying on that surface, along which distance between two points is as small as possible. On a plane, a geodesic is a straight line. Determine equations of geodesics on right circular cylinder. (CO3, K3)

18. (a) Find the Hankel transform of the derivative of a function. (CO3, K3)

Or

(b) State and prove Parseval's theorem. (CO3, K3)

19. (a) Solve the Fredholm integral equation $g(s) = 1 + \lambda \int_0^1 (1 - 3st)\, g(t)\, dt$ and evaluate the resolvent kernel. (CO4, K4)

Or

(b) Solve the integral equation $g(s) = f(s) + \lambda \int_0^1 (s+t)\, g(t)\, dt$ and find the eigenvalues. (CO4, K4)

20. (a) Solve the Fredholm integral equation $g(s) = 1 + \lambda \int_0^1 (1 - 3st)\, g(t)\, dt$ and evaluate the resolvent kernel. (CO5, K5)

Or

(b) State and prove Fredholm's second theorem. (CO5, K5)

———————

R1783

Sub. Code
511518

M.Sc. DEGREE EXAMINATION, NOVEMBER – 2024

Third Semester

Mathematics

Elective – OPTIMIZATION TECHNIQUES

(CBCS – 2022 onwards)

Time : 3 Hours Maximum : 75 Marks

Part A ($10 \times 1 = 10$)

Answer **all** the following objective type questions by choosing the correct option.

1. Which of the following is a direct search method in optimization? (CO1,K1)

   (a) Gradient Descent

   (b) Nelder-Mead method

   (c) Conjugate Gradient Method

   (d) Newton's Method

2. The Kuhn-Tucker conditions are a generalization of which method for solving constrained optimization problems. (CO1,K1)

   (a) Newton's Method

   (b) Gradient Descent

   (c) Lagrange Multipliers

   (d) Simplex Method

3. Which algorithm is commonly used to solve quadratic programming problems? (CO2,K2)

(a) Simplex method

(b) KKT conditions

(c) Newton's method

(d) Gradient descent

4. What is the primary difference between linear programming and quadratic programming? (CO2,K1)

(a) Linear programming deals with linear constraints, while quadratic programming deals with non-linear constraints

(b) Linear programming has a linear objective function, while quadratic programming has a quadratic objective function

(c) Linear programming uses the Simplex method, while quadratic programming uses the Newton method

(d) Linear programming can only minimize objectives, while quadratic programming can both minimize and maximize.

5. The conjugate gradient method is particularly useful for: (CO3,K1)

(a) Solving unconstrained optimization problems with a small number of variables

(b) Solving large-scale unconstrained optimization problems

(c) Problems where the objective function is not differentiable

(d) Problems with numerous constraints

6. Which of the following methods is typically used for finding a local minimum of a differentiable function in unconstrained optimization? (CO3,K1)

   (a) Simplex method

   (b) Gradient descent

   (c) Lagrange multipliers

   (d) Two-phase method

7. The Wolfe conditions are primarily used in: (CO4,K2)

   (a) Unconstrained optimization

   (b) Constrained optimization with equality constraints only

   (c) Line search methods within optimization algorithms

   (d) Integer programming

8. The feasible region of a constrained optimization problem is defined as: (CO4,K1)

   (a) The set of all possible objective function values

   (b) The set of all points that satisfy the constraints

   (c) The set of all points that minimize the objective function

   (d) The set of all points that maximize the objective function

9. Gomory cuts are used in which method to solve Integer Programming problems? (CO4,K1)

   (a) Branch and Bound

   (b) Cutting Plane method

   (c) Dynamic Programming

   (d) Simplex method

10. In the Branch and Bound method, what does the 'branching' process involve? (CO5,K2)

(a) Dividing the problem into smaller subproblems by fixing the values of some integer variables

(b) Combining smaller subproblems into a larger problem

(c) Relaxing the constraints to make the problem linear

(d) Using gradient information to find the optimum solution

Part B $(5 \times 5 = 25)$

Answer **all** the following questions not more than 500 words each.

11. (a) State the necessary and sufficient conditions for the minimum of a function $f(x)$. (CO1,K2)

Or

(b) Write the Taylor's series expansion of a function $f(x)$. (CO1,K2)

12. (a) Find the solution of the following LP problem graphically: (CO2,K2)

Minimize $f = 3x_1 + 2x_2$

Subject to $8x_1 + x_2 \geq 8;$

$2x_1 + x_2 \geq 6;$

$x_1 + 3x_2 \geq 6;$

$x_1 + 6x_2 \geq 8;$

$x_1 \geq 0,\ x_2 \geq 0$

Or

(b) Solve the following system of equations using pivot operations: (CO2,K3)

$6x_1 - 2x_2 + 3x_3 = 11;$

$4x_1 + 7x_2 + x_3 = 21;$

$5x_1 + 8x_2 + 9x_3 = 48.$

13. (a) Find a suitable scaling (or transformation) of variables to reduce the condition number of the Hessian matrix of the following function to 1: (CO3,K2)

$$f(x_1, x_2) = 6x_1^2 - 6x_1x_2 + 2x_2^2 - x_1 - 2x_2.$$

Or

(b) Minimize $f(x_1, x_2) = x_1 - x_2 + 2x_1^2 + 2x_1x_2 + x_2^2$ using random walk method from the point $X_1 = \begin{Bmatrix} 0 & 0 \\ 0 & 0 \end{Bmatrix}$ with a starting step length of $\lambda = 1.0$. Take $\varepsilon = 0.05$ and $N = 100$. (CO3,K3)

14. (a) Consider the problem: (CO4,K3)

Minimize $f(x) = x^2 - 10x - 1$,

Subject to $1 \le x \le 10$.

Plot the contours of the φ_k function using the linear extended interior penalty function method.

Or

(b) Derive the necessary conditions of optimality and find the solution for the following problem: (CO4,K3)

Minimize $f(X) = 5x_1x_2$,

Subject to $25 - x_1^2 - x_2^2 \ge 0$.

15. (a) Solve the following LP problem using the branch-and-bound method: (CO5,K3)

Maximize $f = 3x_1 + 4x_2$,
Subject to

$$7x_1 + 11x_2 \leq 88,$$
$$3x_1 - x_2 \leq 12,$$
$$x_1 \geq 0,\ x_2 \geq 0,$$
$$x_i = \text{integer},\ i = 1, 2.$$

Or

(b) Solve the following problem using Gomory's cutting plane method: (CO5,K3)

Maximize $f = 6x_1 + 7x_2$,

Subject to $7x_1 + 6x_2 \leq 42$;
$5x_1 + 9x_2 \leq 45$;
$x_1 - x_2 \leq 4$,
$x_i \geq 0$ and integer, $i = 1, 2$.

Part C (5 × 8 = 40)

Answer **all** the questions not more than 1000 words each.

16. (a) State the Kuhn–Tucker conditions. (CO1,K2)

Or

(b) In a submarine telegraph cable the speed of signaling vanes as $x^2 \log(1/x)$, where x is the ratio of the radius of the core to that of the covering. Show that the greatest speed is attained when this ratio is $1:\sqrt{e}$. (CO1,K2)

17. (a) Solve LP problem by the revised simple method. (CO2,K2)

Minimize $f = -5x_1 + 2x_2 + 5x_3 - 3x_4$

$2x_1 + x_2 - x_3 = 6;$

Subject to $3x_1 + 8x_3 + x_4 = 7;$

$x_i \geq 0 \; i = 1 \text{ to } 4.$

Or

(b) Solve by quadratic programming: (CO2,K3)

Minimize

$f(X) = 3x_1^2 + 2x_2^2 + 5x_2^3 - 4x_1x_2 - 2x_1x_3 - 2x_2x_3$

Subject to $3x_1 + 5x_2 + 2x_3 \geq 10;$

$3x_1 + 5x_3 \leq 15,$

$x_i \geq 0, \; i = 1, 2, 3.$

18. (a) Prove that the gradient vector represents the direction of steepest ascent. (CO3,K2)

Or

(b) Prove that the maximum rate of change off at any point X is equal to the magnitude of the gradient vector at the same point. (CO3,K3)

19. (a) An open cylindrical vessel is to be constructed to transport 80m^3 of grain from a warehouse to a factory. The sheet metal used for the bottom and sides cost \$80 and \$10 per square meter, respectively. If it costs \$1 for each round trip of the vessel, find the dimensions of the vessel for minimizing the transportation cost. Assume that the vessel has no salvage upon completion of the operation. (CO4,K3)

Or

(b) An automobile manufacturer needs to allocate a maximum sum of $\$2.5 \times 10^6$ between the developments of two different car models. The profit expected from both the models is given by $x_1^{1.5} x_2$, where x_i denotes the money allocated to model $(i = 1, 2)$. Since the success of each model helps the other, the amount allocated to the first model should not exceed four times the amount allocated to the second model. Determine the amounts to be allocated to the two models to maximize the profit expected. (CO4,K3)

20. (a) Convert the following integer quadratic problem into a zero – one linear programming problem: (CO5,K3)

Minimize $f = 2x_1^2 + 3x_2^2 + 4x_1x_2 + 6x_1 - 3x_2$,

Subject to $x_1 + x_2 \le 1$;

$2x_1 + 3x_2 \le 4$,

$x_1, x_2 \ge 0$, integers.

Or

(b) Solve the following mixed integer programming problem using a graphical method: (CO5,K3)

Minimize $f = 4x_1 + 5x_2$,

Subject to $10x_1 + x_2 \ge 10$;

$5x_1 + 4x_2 \ge 20$;

$3x_1 + 7x_2 \ge 21$;

$x_1 + 12x_2 \ge 12$,

$x_1 \ge 0$ and integer $x_2 \ge 0$.

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